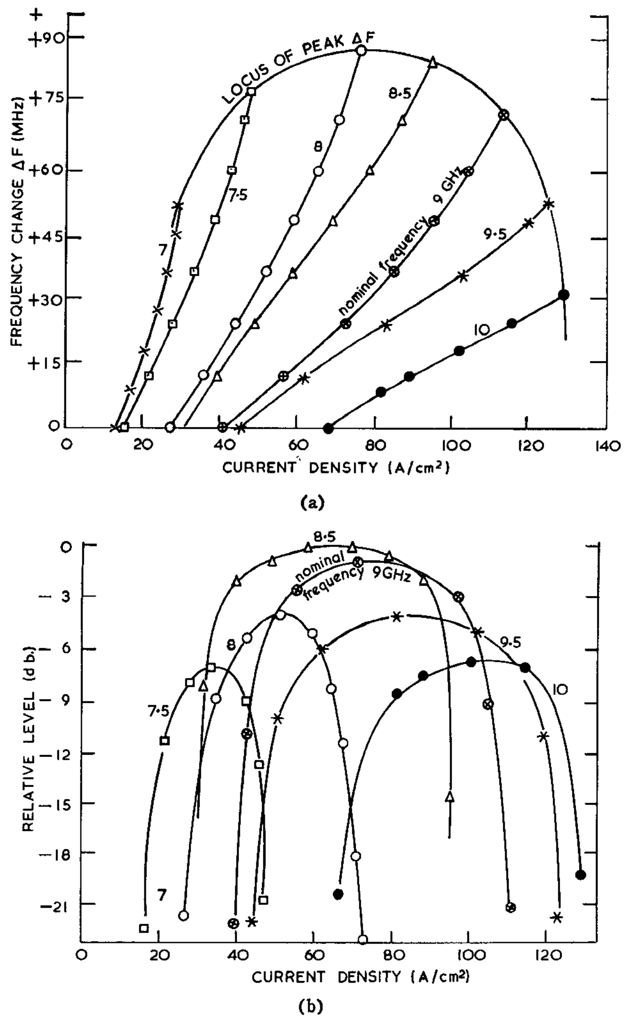

Fig. 4. (a) Measured oscillation frequency change of the electronic tuning versus bias current density (area $=1.2\times10^{-4}$ cm²). (b) Measured oscillation power of the electronic tuning versus bias current density (maximum power $=1.2$ mW).

PITT diode has been investigated at X band. A graphical method of predicting the electronic tuning performance of the oscillator using the results of the dynamic impedance has been outlined. This has been verified experimentally. Therefore, it has been shown to be possible to design the linearity, sensitivity, and level of response of electronic tuning from the knowledge of impedance measurements.[1]

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Anisotropy in Alumina Substrates for Microstrip Circuits

J. H. C. van Heuven and T. H. A. M. Vlek

Abstract—**By determining up to 16 GHz the effective dielectric constant for microstrip lines with various strip widths, it has been found that anisotropy in alumina substrates considerably affects the wavelength and is responsible for most of the discrepancy between theory and experiments.**

In the static theory originally presented by Wheeler [1], the concept of the effective dielectric constant (DC) (ϵ_{eff}) for microstrip lines has been introduced, assuming TEM wave propagation. Early experiments showed the existence of dispersion in microstrip lines. Numerous measurements of dispersion have been reported, mainly on 50-Ω lines on alumina substrates. The discrepancies, observed sometimes between experimental results, could not be explained by the inaccuracy of the measurements. The discrepancies can be partly explained by the difference in the relative DC (ϵ_r) of the substrate [3], [4]. During our measurements of ϵ_{eff} with various strip widths on alumina substrates, we found another cause for the discrepancy between static theory and measured data: anisotropy in the alumina substrates. Anisotropy is a well-known property of sapphire, i.e., a single crystal of aluminum oxide. In polycrystalline alumina a preferred orientation of the crystallites can sometimes be observed. In sapphire, the DC (ϵ_r) in the direction of the optic axis is 10.55 according to [5] or 11.5 according to [6]. For a few synthetic sapphire substrates we found $\epsilon_r=11.7$ (±1 percent), for which the supplier specified 10.55. Perpendicular to the optic axis, $\epsilon_r=8.6$ [5] or 9.5 [6]. For alumina without a preferred orientation, 9.7 is given in [5], which is the same value as specified by the supplier.

In this short paper experiments are described, showing clearly the effect of anisotropy on ϵ_{eff} of microstrip lines on alumina. For comparison, results with isotropic materials: fused quartz and nonmagnetic ferrite, having a lower (3.78) and a higher (13.9) ϵ_r than alumina, are given. The relevant properties of the substrates used are summarized in Table I. The values of ϵ_r are derived from capacitance measurements at 100 kHz.

A simple determination of ϵ_{eff} is obtained from capacitance measurements of microstrip line lengths with various strip widths at a sufficiently low frequency (100 kHz). Then ϵ_{eff} is defined as

$$\epsilon_{eff} = \frac{C_l}{C_0}$$

where C_l denotes the measured capacity of the line and C_0 the capacity of the line for substrates with $\epsilon_r=1$. The latter cannot be measured, but is derived from the characteristic impedance of a line for substrates with $\epsilon_r=1$ as given in [2]. Since $Z_0=(L/C_0)^{1/2}$ and $c=3.10^8$ m/s $=(LC_0)^{-1/2}$, ϵ_{eff} can be written as

$$\epsilon_{eff} = C_l\cdot c\cdot Z_0.$$

This value, denoted by ϵ_{eff} *(dc)*, should equal that obtained from the static theory [2], denoted by ϵ_{eff} *(st)*, within the accuracy of the approximation and that of the measurement (about 1 percent).

A second method was used for determining dispersion and for verifying the data ϵ_{eff} *(dc)* obtained from capacitance measurements. By measuring the resonance frequency of transmission-type resonators, consisting of lengths of coupled microstrip lines, the wavelength can be determined and ϵ_{eff} is found as

$$\epsilon_{eff}(w) = \left(\frac{\lambda_0(w)}{\lambda_g(w)}\right)^2$$

where $\lambda_0(w)$ and $\lambda_g(w)$ represent the wavelength in free space and in the microstrip line, respectively, at an angle frequency w. $\epsilon_{eff}(w)$ depends on frequency, indicating dispersion. Corrections Δl for the fringe fields at the open ends of the resonators are derived from measurements with resonant line lengths $l_1\approx n_1\lambda_g(w_1)/2$ and $l_2\approx n_2\lambda_g(w_2)/2$ where $n_{1,2}$ is a whole number and $w_1\approx w_2$ [7]. The correction follows

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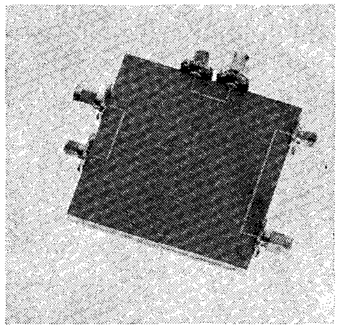

Fig. 1. Resonant microstrip lines on fused quartz with different lengths for calculation on the fringe field at the open ends.

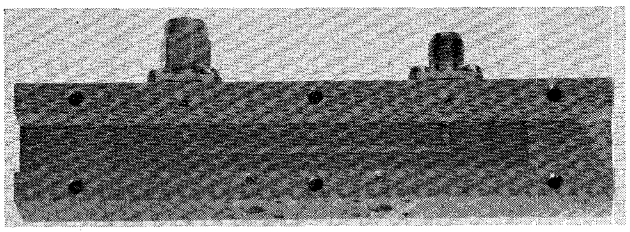

Fig. 2. Resonant microstrip line on fused quartz mounted in a *Ku*-band waveguide with upper wall removed.

TABLE I

	Substrate Thickness h (mm)	Dieletric Constant ϵ_r (at 100 kHz)	Surface Roughness (μm)
Fused quartz	0.50 ±0.005	3.78	0.4–0.6
Alumina (Alsimag 772)	0.635±0.01	10.8±0.1	0.2–0.4
Nonmagnetic ferrite	0.55 ±0.002	13.9±0.1	<0.05 polished

from

$$\Delta l = \frac{1}{2}\left|\frac{n_2 w_1 l_1 - n_1 w_2 l_2}{n_2 w_1 - n_1 w_2}\right|.$$

A fused quartz substrate comprising three line lengths for the resonance measurements is shown in Fig. 1. Within the estimated accuracy (20 percent) of Δl, no dependency on frequency could be observed. Taking the correction Δl into account, $\epsilon_{eff}(w)$ has been measured between 2 and 12 GHz (16 GHz for 50-Ω lines). From these data the effective dielectric constant at zero frequency has been derived by extrapolation and will be denoted by $\epsilon_{eff}(0)$. The values of $\epsilon_{eff}(dc)$ and $\epsilon_{eff}(0)$ should be equal. The reproducibility is better than 0.1 percent and the estimated absolute accuracy is better than 1 percent. With the line resonators mounted in a *Ku*-band waveguide (Fig. 2), with a cutoff frequency of 15 GHz, the difference between the values of $\epsilon_{eff}(w)$ measured in and out of the waveguide was less than 0.4 percent, which is within the limits of the estimated accuracy.

$\epsilon_{eff}(dc)$ obtained from capacitance measurements on fused quartz and ferrite are compared in Fig. 3 with the curves calculated from the static theory given by Schneider [2]. These formulas are preferred to those presented by Wheeler [1] (dotted curves in Fig. 3) because of the ambiguity in the approximation for narrow and wide strips. The agreement between experiment and theory is satisfactory. For alumina the discrepancy is obvious, as shown in Fig. 4. For wide strips, experimental values of the effective dielectric constant agree well with the predicted curve for $\epsilon_r = 10.8$, the measured value, taking into account the error in the approximation used in the static theory and the accuracy of the measurement. However, for decreasing strip width, $\epsilon_{eff}(1c)$ decreases rapidly to the curve belonging to $\epsilon_r = 9.7$, the value specified by the supplier.

Resonance measurements were carried out only on fused quartz and alumina because the supply of ferrite substrates was limited. The corrections Δl for fringe fields are summarized in Table II and agree well with the predicted values given in [8]. The data for $\epsilon_{eff}(w)$ are

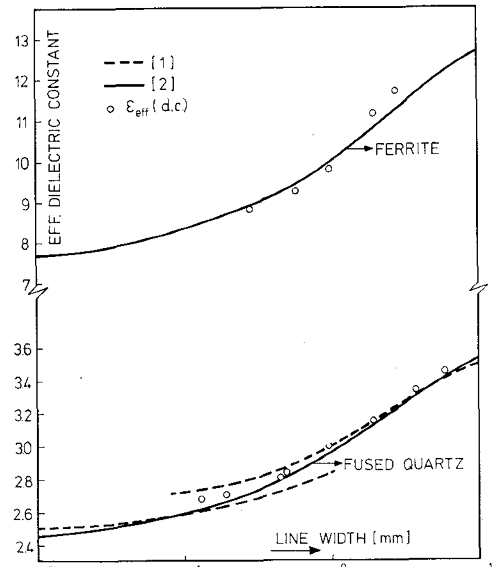

Fig. 3. Effective dielectric constant at 100 kHz of microstrip lines on fused quartz and nonmagnetic ferrite compared with curves derived from [1] and [2].

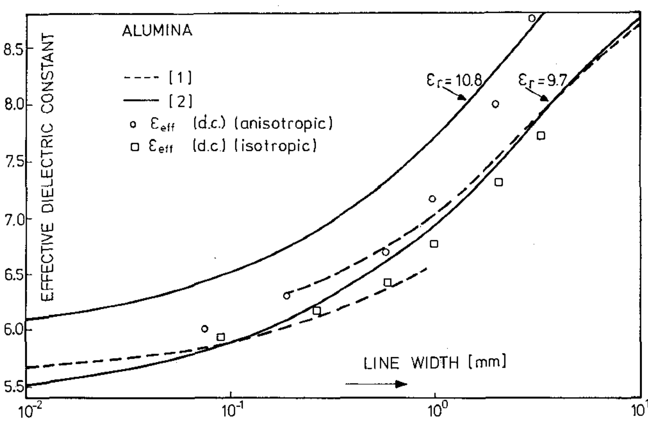

Fig. 4. Effective dielectric constant at 100 kHz of microstrip lines on anisotropic and isotropic alumina compared with curves from [1] and [2].

TABLE II

Approximate Characteristic Impedance (Ω)	Fused Quartz Δl (mm)	Fused Quartz w (mm)	Alumina Δl (mm)	Alumina w (mm)
25	0.26	3.00	0.21	2.00
40	0.24	1.50	0.18	0.90
50	0.20	1.00	0.18	0.58
70	0.15	0.50	0.15	0.20
100	0.14	0.15	0.14	0.07

summarized in Table III. For fused quartz, the extrapolated values $\epsilon_{eff}(0)$ agree very well with $\epsilon_{eff}(st)$. For alumina, a similar decrease with decreasing line width is found in $\epsilon_{eff}(0)$ as in $\epsilon_{eff}(dc)$.

This effect in alumina can be explained by assuming anisotropy in the substrates. For wide lines the main parts of the electric field lines are directed perpendicular to the surface of the substrate, but for narrow lines the component of the fringe field parallel to the surface cannot be neglected. This implies that crystallites align themselves perpendicular with the optic axis to the substrate surface. A strong preferential orientation could indeed be demonstrated by X-ray diffraction analysis. This anisotropy is also responsible for the differences found in ϵ_r. In order to substantiate this statement, a few substrates were selected from a different supplier with a lower measured ϵ_r (9.9). No preferred orientation could be observed by X-ray diffraction analysis and the values for $\epsilon_{eff}(dc)$ showed a good agree-

TABLE III

Strip Width (mm)	ϵ_{eff} (*st*)	ϵ_{eff} (0)	$\epsilon_{eff}(w)$at 4 GHz	8 GHz	12 GHz	16 GHz
Fused Quartz						
3.00	3.25	3.26	3.28	3.31	3.36	
1.50	3.06	3.09	3.10	3.12	3.15	
1.00	2.96	2.97	2.98	2.99	3.02	3.06
0.50	2.81	2.86	2.87	2.88	2.89	
0.15	2.63	2.66	2.67	2.67	2.68	
Alumina						
2.00	8.30	7.85	8.06	8.31	8.60	
0.90	7.63	7.07	7.27	7 40	7.60	
0.58	7.33	6.86	6.93	7 11	7.31	7.52
0.20	6.76	6.25	6.30	6.40	6.50	
0.07	6.41	6.00	6.00	6.10	6.20	

ment with the static theory (Fig. 4). The orientation of the crystallites depends on the manufacturing processes and is not necessarily constant over the substrate. The anisotropy in alumina substrates can be inconvenient, especially when used for circuits comprising narrow-band filters and when experimentally verifying theories.

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The Green's Function for Poisson's Equation in a Two-Dielectric Region

ANTONIO F. DOS SANTOS AND VICTOR R. VIEIRA

***Abstract*—The validity of the reciprocity relation satisfied by the Green's function for Poisson's equation in a two-dielectric region is briefly discussed.**

INTRODUCTION

In calculating the parameters of a stripline by variational techniques it is often necessary to determine first a Green's function for the two-dimensional Poisson's equation in the region bounded by the two conductors [1], [2]. Contrary to the case of a single dielectric [2], the symmetry properties of the Green's function in a two-dielectric region do not appear to have been dealt with in the literature.

The aim of this short paper is to point out that the reciprocity relation satisfied by the Green's function in the latter case is only valid for a specific form of the right-hand side of the differential equation defining the Green's function. Only the Green's function subject to Dirichlet boundary conditions will be considered.

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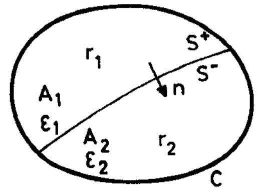

Fig. 1.

RECIPROCITY RELATION

Let $G(r, r_0)$ be the function satisfying the following conditions in the two-dielectric region $A = A_1 \cup A_2$ (see Fig. 1):

$$\nabla^2 G = -\frac{1}{\epsilon}\delta(r - r_0), \qquad r \in A \tag{1a}$$

$$G = 0, \qquad r \in C \tag{1b}$$

$$G|_{S^+} = G|_{S^-} \tag{1c}$$

$$\epsilon_1 \frac{\partial G}{\partial n}\bigg|_{S^+} = \epsilon_2 \frac{\partial G}{\partial n}\bigg|_{S^-} \tag{1d}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y_2}.$$

Applying the Green's identity [2] separately to regions A_1 and A_2, in which G and its first-order partial derivatives are continuous with the only exception of the source point $(r = r_0)$, the following equations are readily obtained:

$$\frac{1}{\epsilon_1} G_2|_{r=r_1} = \int_S \left(G_1 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial G_1}{\partial n}\right)_{r \in S^+} dl \tag{2a}$$

$$-\frac{1}{\epsilon_2} G_1|_{r=r_2} = -\int_S \left(G_1 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial G_1}{\partial n}\right)_{r \in S^-} dl \tag{2b}$$

where G_1 and G_2 denote $G(r, r_1)$ and $G(r, r_2)$, respectively. Substitution of (1c) and (1d) into these equations yields the reciprocity relation

$$G(r_1, r_2) = G(r_2, r_1) \tag{3}$$

which shows that the Green's function is symmetric in its two arguments. Examination of (2a) and (2b) shows, however, that if the RHS of (1a) is simply $\delta(r - r_0)$, the reciprocity relation (3) no longer holds. In fact, it can be shown without much difficulty that in this case, the function G is not the true Green's function for Poisson's equation subject to the boundary conditions (1c) and (1d).

Finally we note that in view of relation (3), to determine G completely it is sufficient to consider the case where the source point is located in one of the two-dielectric regions, e.g., A_1.

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"Unfolding" the Lange Coupler

RAYMOND WAUGH AND DAVID LACOMBE

The broad-band microstrip quadrature coupler described by Lange [1] is shown in Fig. 1(a). True quadrature coupling over an octave is realized as a consequence of the interdigital coupling section which compensates for even- and odd-mode phase velocity dispersion over the wide frequency range. A power-split variation between the direct and coupled ports, ports 3 and 4, respectively, in Fig. 1(a), of

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